Complete the Square

We can manipulate a quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ to write it as the square of a binomial equal to a constant. First look at the relationship between a perfect square trinomial and its factored form.

Perfect Square Trinomial	Factored Form
$x^2 + 10x + 25$	$(x + 5)^2$
$t^2 - 6t + 9$	$(t - 3)^2$
$p^2 - 14p + 49$	$(p - 7)^2$

For a perfect square trinomial with a leading coefficient of 1, the constant term is the square of one-half the linear term coefficient. For example:

$$x^2 + 10x + 25 \qquad \left[\tfrac{1}{2}(10)\right]^2 = 25$$

In general, an expression of the form $x^2 + bx + n$ is a perfect square trinomial if $n = \left(\frac{1}{2}b\right)^2$. The process to create a perfect square trinomial is called **completing the square.**

Completing the Square

Determine the value of n that makes the polynomial a perfect square trinomial. Then factor the expression as the square of a binomial.

a. $x^2 + 18x + n$ **b.** $x^2 - 13x + n$ **c.** $x^2 + \frac{4}{7}x + n$

Solution:

a. $x^2 + 18x + n$ — To find n, take $\frac{1}{2}$ of 18, and square the result.

$= x^2 + 18x + 81$ — $n = \left[\frac{1}{2}(18)\right]^2 = [9]^2 = 81$

$= (x + 9)^2$ — Factor.

b. $x^2 - 13x + n$ — To find n, take $\frac{1}{2}$ of -13, and square the result.

$= x^2 - 13x + \frac{169}{4}$ — $n = \left[\frac{1}{2}(-13)\right]^2 = \left[-\frac{13}{2}\right]^2 = \frac{169}{4}$

$= \left(x - \frac{13}{2}\right)^2$ — Factor.

c. $x^2 + \frac{4}{7}x + n$ — To find n, take $\frac{1}{2}$ of $\frac{4}{7}$, and square the result.

$= x^2 + \frac{4}{7}x + \frac{4}{49}$ — $n = \left[\frac{1}{2}\left(\frac{4}{7}\right)\right]^2 = \left[\frac{2}{7}\right]^2 = \frac{4}{49}$

$= \left(x + \frac{2}{7}\right)^2$ — Factor.

Completing the Square and Solving a Quadratic Equation

Solve the equation by completing the square and applying the square root property. $x^2 - 3 = -10x$

Solution:

$x^2 - 3 = -10x$	Write the equation in the form $ax^2 + bx + c = 0$.
$x^2 + 10x - 3 = 0$	**Step 1:** Notice that the leading coefficient is already 1.
$x^2 + 10x + \_\_\_ = 3 + \_\_\_$	**Step 2:** Add 3 to both sides to isolate the variable terms.
$x^2 + 10x + 25 = 3 + 25$	**Step 3:** Add $\left[\frac{1}{2}(10)\right]^2 = [5]^2 = 25$ to both sides.
$(x + 5)^2 = 28$	Factor.
$x + 5 = \pm\sqrt{28}$	**Step 4:** Apply the square root property and solve for x.
$x = -5 \pm 2\sqrt{7}$	Both solutions check in the original equation.
$\{-5 \pm 2\sqrt{7}\}$	Write the solution set.

Solve the equation by completing the square and applying the square root property. $-2x^2 - 3x - 5 = 0$

Solution:

$-2x^2 - 3x - 5 = 0$	The equation is in the form $ax^2 + bx + c = 0$.
$\frac{-2x^2}{-2} - \frac{3x}{-2} - \frac{5}{-2} = \frac{0}{-2}$	**Step 1:** Divide by the leading coefficient, -2.
$x^2 + \frac{3}{2}x + \frac{5}{2} = 0$	The new leading coefficient is 1.
$x^2 + \frac{3}{2}x + \_\_\_ = -\frac{5}{2} + \_\_\_$	**Step 2:** Subtract $\frac{5}{2}$ from both sides to isolate the variable terms.
$x^2 + \frac{3}{2}x + \frac{9}{16} = -\frac{5}{2} + \frac{9}{16}$	**Step 3:** Add $\left[\frac{1}{2}\left(\frac{3}{2}\right)\right]^2 = \left[\frac{3}{4}\right]^2 = \frac{9}{16}$ to both sides.
$\left(x + \frac{3}{4}\right)^2 = -\frac{40}{16} + \frac{9}{16}$	Factor.
$\left(x + \frac{3}{4}\right)^2 = -\frac{31}{16}$	
$x + \frac{3}{4} = \pm\sqrt{-\frac{31}{16}}$	**Step 4:** Apply the square root property and solve for x.
$x = -\frac{3}{4} \pm i\frac{\sqrt{31}}{4}$	Simplify the radical. The solutions both check in the original equation.
$\left\{-\frac{3}{4} \pm \frac{\sqrt{31}}{4}i\right\}$	Write the solution set.